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On possibilistic and probabilistic uncertainty assessment of power flow problem: A review and a new approach

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A B S T R A C T

As energy resource planning associated with environmental consideration are getting more and more challenging all around the world, the penetration of distributed energy resources (DERs) mainly those harvesting renewable energies (REs) ascend with an unprecedented rate. This fact causes new uncertainties to the power system context; ergo, the uncertainty analysis of the system performance seems necessary. In general, uncertainties in any engineering system study can be represented probabilistically or possibilistically. When sufficient historical data of the system variables is not available, a probability density function (PDF) might not be defined, while they must be represented in another manner i.e. possibilistically. When some of system uncertain variables are probabilistic and some are possibilistic, neither the conventional pure probabilistic nor pure possibilistic methods can be implemented. Hence, a combined solution methodology is needed. This paper proposes a new analytical probabilistic- possibilistic tool for the power flow uncertainty assessment. The proposed methodology is based upon the evidence theory and joint propagation of possibilistic and probabilistic uncertainties. This possibilistic–probabilistic formulation is solved in an uncertain power flow (UPF) study problem.

Contents

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1. Introduction

Power flow (PF) assessment of electrical power systems is of essential tools for both system operation and planning. Deterministic PF analysis requires precise values of demand, generation, and network conditions. More and more uncertainties emerge in world-wide power systems as a by-product of restructuring on one hand and the unprecedented penetration of uncertain distributed energy resources (DERs) especially renewable energies (REs) on the other. Distributed generations (DGs) are defined as electric resources interconnected to the distribution networks [1]. The implementation of DGs has many technical [2], socio-economic [3] and environmental [4] benefits but they, especially those harvesting REs, bring about additional uncertainties in the system performance that must be studied clearly [5].

In such uncertain conditions, deterministic PF calculation cannot precisely reveal the state of system; therefore, uncertainty point of view analysis would be of significant interest. Generally, probabilistic power flow (PPF), aiming to assess bus voltages and line flows in response to the uncertain system parameters, is recognized as a desirable tool in both power system planning and operation. This tool informs system engineers over the future system conditions and helps them in the decision making process in such an uncertain environment. Probabilistic methods are of the main approaches being dealt with such uncertainties, where they can be applied when sufficient historical data regarding uncertain variables is available. Usually, there are some uncertain variables that their historical data are incomplete or obtained by experience; hence, the possibilistic theory might be used to model their uncertainty associated with imperfection as well as imprecision. This is not all the thing, consider the case in which some variables are probabilistic and some are possibilistic which is a common case in real world. In such circumstance, a joint propagation of probabilistic and possibilistic uncertainties would be fruitful. This work is motivated by the joint propagation of these kinds of uncertainties in the PF problem, the so-called uncertain power flow (UPF) study.

So far, various techniques have been used for uncertainty analysis of engineering systems. In the early seventies and for the first time, the system load uncertainty was considered in a probabilistic analysis of PF problem [6]. Till now, many probabilistic approaches have been suggested. Monte Carlo simulation (MCS) is the frequently used simulation-based probabilistic method. In [7], the effects of wind power integration on the power system PF problem solutions are studied by the MCS. Generally, MCS is recognized as a system-size independent technique; however, it may have a long execution time [8]. In [9] and [10], the PPF problem was studied by the combined Cumulant and Gram–Charlier expansion theory. In [11], the integration of wind farms in AC-PPF problem was studied. In [12], in order to reduce the computational burden, a discrete frequency domain convolution technique using fast Fourier transformation (FFT) is proposed. In [13], a point estimation-based method (PEM) was developed for the PPF problem. In [5], the Unscented Transformation (UT) was used for probabilistic studies of the PF problem.

As noted before, probabilistic methods are applicable when sufficient historical data about uncertain variables or their PDF are available. Generally, there are some uncertain variables in any system that their PDF are not available due to variety of reasons such as imprecise historical data, privacy reasons and so forth. In this situation, the possibility theory can be utilized to cope with this kind of uncertainties. In [14–16], fuzzy modeling has been used to reflect the uncertainties in the PF problem. As a general rule, any engineering system has different types of uncertainties especially in its input variables. In real world, some uncertain variables are probabilistic and some are possibilistic, in which neither pure probabilistic nor pure possibilistic techniques can be implemented alone but a new combinatorial solution is needed. Indeed, a joint propagation of probabilistic and possibilistic uncertainties must be carried out [17]. Hence, a possibilistic–probabilistic tool to evaluate the uncertainty of engineering systems is of huge interest. The main contribution of this work is to develop a new methodology for UPF studies using evidence theory to joint propagation of probabilistic and possibilistic uncertainties applied to power system PF studies, taking into account uncertainties associated with the load, wind, and solar power generation as probabilistic uncertain parameters and the gas turbine distributed generation (DG) and electric vehicles (EVs) as uncertain possibilistic variables. To this end, the uncertainties are first modeled appropriately; thereafter, the proposed method is devised by adaptation of evidence theory to the problem. In the following, the proposed methodology is applied on test systems including a 9-bus and a realistic 574-node distribution system. Note that in the recent published works e.g. [17], the MCS has been used for probabilistic frame work in the joint algorithm which itself suffers from computational effort. On the contrary, in this work, the Unscented Transformation (UT) method whose application in the PPF problem has been confirmed in [5] is proposed as probabilistic frame work of the joint algorithm. The proposed method has greatly reduced the computational burden while remaining the desired accuracy compared to the MCS-based method proposed in [17].

The remainder of the paper is structured as follows. Section 2 presents the PF and UPF formulation, briefly. Section 3 discusses about probabilistic uncertainty modeling and some of its approaches. It also introduces the Unscented Transformation (UT) method. In Section 4, the possibilistic uncertainty modeling is given. In Section 5, the uncertainties in the PF problem and their modeling are introduced. Section 6 presents the evidence theory and the proposed algorithm for joint propagation of probabilistic and possibilistic uncertainties. Simulation results will come in

Nomenclature

D	index of loads at each bus, running from 1 to D_i
G	index of generating units at each bus, running from 1 to G_i
i,j	indices for buses, running from 1 to N_B
t	index of UT samples, running from 1 to n
a_l	a parameter of fuzzy trapezoidal membership function
$a_{\max}$	a parameter of fuzzy trapezoidal membership function
$a_{\min}$	a parameter of fuzzy trapezoidal membership function
a_u	a parameter of fuzzy trapezoidal membership function
C_i^{DG}	capacity of DG installed at bus i [MW]
C_i^{EV}	capacity of a given EV at bus i [kW]
C_w	the weibull scale parameter of wind speed [m/s]
k	the number of uncertain probabilistic variables
k_w	the weibull shape parameter of wind speed
N	total number of uncertain variables
$\mathbf{P}_{DER}$	vector of DER's active power generation [p.u.]
$\mathbf{P}_G$	vector of generators' active power generation [p.u.]
$\mathbf{P}_L$	vector of loads' active power consumption [p.u.]
$\mathbf{P}_{XX}$	covariance matrix of probabilistic input variables vector
$\mathbf{P}_{YY}$	covariance matrix of probabilistic output variables vector
P_i^{DG}	power generation of DG installed at bus i [MW]
P_i^{EV}	power generation/consumption of a given EV located at bus i[kW]
P_i^{net}	net active power injection at bus i [p.u.]
P_{rs}	SCG rated power [MW]
P_{rw}	WTG rated power [MW]
P_{slack}	the active power generation of slack bus generator [p.u.]
P_{SCG}	the active power generated by SCG [MW]
P_{WTG}	the active power generated by WTG [MW]
$\mathbf{Q}_{DER}$	vector of DER's reactive power generation [p.u.]
$\mathbf{Q}_G$	vector of generators' reactive power generation [p.u.]
$\mathbf{Q}_L$	vector of loads' reactive power consumption [p.u.]
$\mathbf{Q}_{PV}$	vector of reactive power generation at PV buses [p.u.]
$Q_i^{\max-PV}$	upper limit of reactive power of generation unit at PV bus i [p.u.]
$Q_i^{\min-PV}$	lower limit of reactive power of generation unit at PV bus i [p.u.]
Q_i^{net}	net reactive power injection at bus i [p.u.]
Q_{slack}	the reactive power generation of slack bus generator [p.u.].
r	solar radiation [W/m^2]
R_C	a certain radiation point, usually equal to 150 W/m^2
R_{STD}	solar radiation in the standard radiation, usually 1000 W/m^2
S_{ij}	line flow between buses i,j [p.u.]
U	universe of discourse
v	wind speed [m/s]
$\mathbf{V}$	vector of buses' voltage magnitude [p.u.]
V_{cut-in}	wind turbine cut-in speed [m/s]
$V_{cut-out}$	wind turbine cut-out speed [m/s]
V_{rated}	wind turbine rated speed [m/s]
$V_i^{\max-PQ}$	upper limit of voltage at PQ bus i [p.u.]
$V_i^{\min-PQ}$	lower limit of voltage at PQ bus i [p.u.]
W^t	weight associated with the tth sample point of the UT method
x_i^e	the eth sampling of ith probabilistic variable
$\mathbf{X}$	vector of uncertain input variables
$\bar{\mathbf{X}}$	vector of mean value for probabilistic input variables
$\mathbf{x}^t$	the tth sample point of probabilistic uncertain variables
y^e	the transformed eth sampling of probabilistic input variables
$\mathbf{Y}$	vector of uncertain output variables
Y_{ij}	admittance of line connecting nodes i,j [p.u.]
Y^{α}	lower bound of output variable's $\alpha-cut$
$\bar{Y}^{-\alpha}$	upper bound of output variable's $\alpha-cut$
α	a certain value of the membership function
δ	the buses' voltage angle vector [rad]
δ_i	voltage angle at bus i [rad]
θ_{ij}	angle of Y_{ij} [rad]
$\pi_{\tilde{X}}(x)$	the membership degree of x to set$X$
$\Delta\alpha$	the step size of α in the joint algorithm
α_β	beta shape parameter [kW/m^2]
β_β	beta shape parameter [kW/m^2]
Remark 1	a variable or parameter written in bold is a vector form representing the corresponding quantities
Remark 2	a variable, function, set, or any parameter written with a ~ on it represents a possibilistic parameter

Section 7. Finally, Section 8 closes the paper providing the concluding remarks.

2. Power flow (PF) problem

Before introducing the probabilistic or possibilistic PF and its formulation, it will be helpful to recall a general formulation of the deterministic PF problem first.

2.1. PF formulation

Generally speaking, the PF problem is a tool to find the system state and control variables subject to some constraints that can be summarized as the following:

$$P_i^{net} = \sum_{G=1}^{G_i} P_i^G - \sum_{D=1}^{D_i} P_i^D \tag{1}$$

$$Q_i^{net} = \sum_{G=1}^{G_i} Q_i^G - \sum_{D=1}^{D_i} Q_i^D \tag{2}$$

$$P_i^{net} = |V_i| \sum_{j=1}^{N_B} |Y_{ij}||V_j| \cos(\delta_i - \delta_j - \theta_{ij}) \tag{3}$$

$$Q_i^{net} = |V_i| \sum_{j=1}^{N_B} |Y_{ij}||V_j| \sin(\delta_i - \delta_j - \theta_{ij}) \tag{4}$$

It has some inequality constraints, too. The maximum and minimum allowable limits for bus voltages at PQ buses (load buses) and reactive power production of generating units at PV buses (generation buses) appear in its inequality constraints such as

$$V_i^{\min-PQ} \le V_i^{PQ} \le V_i^{\max-PQ} \tag{5}$$

$$Q_i^{\min-PV} \le Q_i^{PV} \le Q_i^{\max-PV} \tag{6}$$

2.2. Uncertain power flow formulation

As a well-known fact, power systems are intrinsically uncertain systems. Accordingly, in an UPF study, it is desired to determine the state of the system as a function of uncertain input variables. Currently, most of the existing uncertainty assessment approaches applied to power systems are developed on the assumption that all types of uncertainties of the power grid can be represented probabilistically, described in terms of their PDF. This type of uncertainty is commonly addressed as aleatory and stochastic randomness due to intrinsic variability of the uncertain variables [18]. The other type of uncertainty exists due to the incomplete or imprecise knowledge and lack of sufficient information about some system parameters which results in inexactness in the system modeling and inevitably in its performance evaluation. This type of uncertainty is usually addressed as epistemic [18]. Probabilistic and possibilistic uncertainties require different mathematical representations and analysis [19]. When there is not enough information to establish the PDF of system uncertain variables, the possibility distribution is used to represent the epistemic uncertainties [20]. Indeed, when some of the input variables are uncertain (probabilistic or possibilistic), (1)–(4) become uncertain. They can be stated as

$$\mathbf{Y} = h(\mathbf{X}) \tag{7}$$

where, $\mathbf{X}$ and $\mathbf{Y}$ are the vector of uncertain input and output variables, respectively. The input vector includes the demand, network conditions, states of generating units, and the power generated by the DERs such as wind and solar farms among others, which can be written as

$$\mathbf{X} = \left[x_1, x_2, \ldots, x_k, \tilde{x}_{k+1}, \ldots, \tilde{x}_N\right]^T = \left[\mathbf{P}_L \mathbf{Q}_L \mathbf{P}_{DER} \mathbf{Q}_{DER} \ldots\right]^T \tag{8}$$

Note that in this notation, the first k variables of the input vector are probabilistic and the last $N-k$ ones are possibilistic. The output vector $\mathbf{Y}$ which its elements are uncertain, can be stated as

$$\mathbf{Y} = [\mathbf{V} \ \ \delta \ \ P_{slack} \ \ Q_{slack} \ \ \mathbf{Q}_{PV} \mathbf{S}_{ij} \ldots]^T \tag{9}$$

The PF output variables may be system state variables (e.g. $\mathbf{V}$ and δ) or control variables (e.g. P_{slack}, Q_{slack},$\mathbf{Q}_{PV}$,…).

3. Probabilistic uncertainty modeling

As a well-known fact, the frequently used approach for uncertainty study is to use the idea of probability theory. So far, various set of approaches have been used for the probabilistic analysis of power systems. These methods can be classified into two categories: simulation-based and analytical methods.

3.1. Simulation based probabilistic methods

Monte Carlo simulation (MCS) is the widely used simulation method. MCS is recognized to be a system-size independent approach and is used when the system is highly nonlinear, complicated or has many uncertain variables [8]. Sequential MCS, pseudo-sequential MCS, and non-sequential MCS are three different types of MCS techniques used for probabilistic uncertainty analysis of engineering systems. The non-sequential MCS is the most efficient and provides comparable accuracy to sequential MCS with less computational burden [20]. The MCS repeats solving a given problem M times: at each eth repetition, a sample vector of uncertain input variables (x_1^e, x_2^e, …, x_N^e) is chosen from the PDFs of the input variables. At the next step, the transformation of input vector i.e. y^e is computed solving the system model.

$$y^e = h(x_1^e,\ x_2^e, \ldots,\ x_N^e) \tag{10}$$

After M repetitions, an approximated estimate of the distribution of the system output variable e.g. the mean and standard deviation (STD) values can be obtained. In this work, the non-sequential MCS is used.

$$\mu_y = \mathrm{Mean}(y^1,\ y^2\ , \ldots,\ y^M) \tag{11}$$

$$\sigma_y = \mathrm{STD}(y^1,\ y^2,\ \ldots,\ y^M) \tag{12}$$

3.2. Analytical probabilistic methods

Analytical methods are generally founded on some mathematical assumptions, simplifications, and have more complex algorithms [21]. The analytical methods can themselves be categorized into two distinct groups. The first group methods are based on linearization and Taylor series expansion such as Gram–Charlier method which attempt to linearize the nonlinear transformation function. The second group methods are based on the intuition that it is easier to approximate a probability distribution function than to approximate a nonlinear transformation function [21]. The heart of the analytical methods lie in how to generate appropriate samples of input variables that can maintain sufficient information about the input variable's PDF. Two of these methods are point estimation method (PEM) [13] and Unscented Transformation (UT) [21–23] among the rest. Stress again that, probabilistic methods can be implemented in situations that sufficient historical data does exist about uncertain variables to construct the PDF of these variables. When such historical data does not exist, the possibilistic uncertainty modeling may be useful. In this work and due to the superiority of the UT method mentioned in [5], this method is used as the probabilistic method.

3.3. Unscented Transformation (UT) method

The UT method was developed to overcome the deficiencies associated with traditional probabilistic methods especially those use the linearization process. It is a reliable method for calculating the statistics of output random variables undergoing a set of nonlinear transformations. The fact that it is easier to approximate a probability distribution than an arbitrary nonlinear function is its foundation [24]. Suppose that $\mathbf{X}$ is a vector of n-dimensional probabilistic uncertain variables which has mean $\overline{\mathbf{X}} = \mathbf{m}$, and covariance $\mathbf{P_{XX}}$. Consider another uncertain variable $\mathbf{Y}$ relating to $\mathbf{X}$ through a nonlinear function such as (10); where, f can be a set of nonlinear functions. The UT method can be used to obtain the mean and covariance of the output variable, $\overline{\mathbf{Y}}$ and $\mathbf{P_{YY}}$, through the following simple steps [21].

Step 1: Obtain $2n+1$ samples or sigma points of input variables through (13)–(15).

$$\mathbf{x}^0 = \mathbf{m} \tag{13}$$

$$\mathbf{x}^k = \mathbf{m} + \left(\sqrt{\frac{n}{1-W^0}\mathbf{P_{XX}}}\right)_k, \quad k = 1, 2, \ldots, n \tag{14}$$

$$\mathbf{x}^{k+n} = \mathbf{m} - \left(\sqrt{\frac{n}{1-W^0}\mathbf{P_{XX}}}\right)_k \quad , \quad k = 1, 2, ..., n \tag{15}$$

Step 2: Calculate the weight associated with each sample using (16)–(18).

$$W^0 = W^0 \tag{16}$$

$$W^k = \frac{1-W^0}{2n}, \quad k = 1, 2, ..., n \tag{17}$$

$$W^{k+n} = \frac{1-W^0}{2n}, \quad k+n = n+1, \ldots, 2n \tag{18}$$

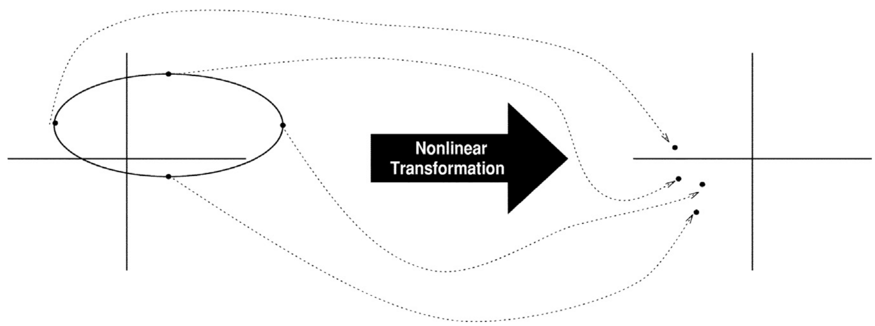

Fig. 1. The principle of the UT method, step 3 [21].

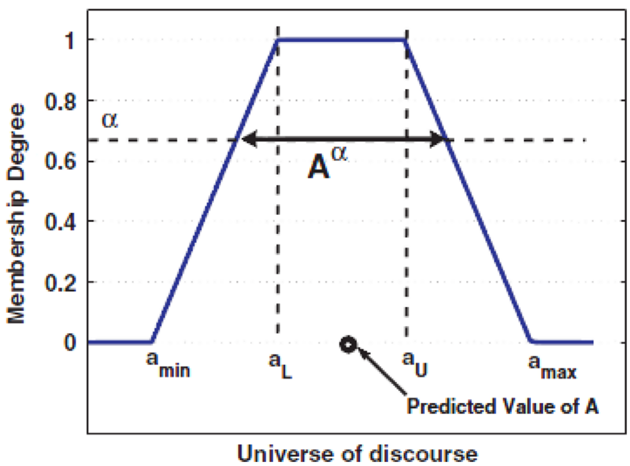

Fig. 2. A fuzzy trapezoidal membership function [17].

Note that the associated weights must meet the condition that

$$\sum_{k=0}^{2n} W^k = 1 \tag{19}$$

In (14) and (15), $(\sqrt{(n/1-W^0)\mathbf{P}_{\mathbf{XX}}})_k$ is the kth row or column of matrix square root of $((n/1-W^0))\mathbf{P}_{\mathbf{XX}}$. The matrix square root of positive definite matrix $\mathbf{P}$ means that there is a matrix $\mathbf{A}=\sqrt{\mathbf{P}}$ such that $\mathbf{P}=\mathbf{A}\mathbf{A}^{\mathbf{T}}$[21].
Here, W^0 is the assigned weight to the point $\overline{\mathbf{X}}=\mathbf{m}$, named as the zeroth point. It controls the location of other points around the mean value of X [21].

Step 3: Feed each sample point to the nonlinear function to yield a set of transformed sample points as

$$\mathbf{y}^k = f(\mathbf{X}^k) \tag{20}$$

It must be emphasized that in the UT method, the nonlinear function is considered as a black box; hence, no simplification or linearization is required. Fig. 1 portrays this procedure.

Step 4: Calculate the mean and covariance of output variable $\mathbf{Y}$ using (21) and (22), respectively.

$$\bar{\mathbf{Y}} = \sum_{k=0}^{2n} \mathbf{W}^k \mathbf{Y}^k \tag{21}$$

$$\mathbf{P}_{\mathbf{YY}} = \sum_{k=0}^{2n} \mathbf{W}^k (\mathbf{Y}^k - \bar{\mathbf{Y}})(\mathbf{Y}^k - \bar{\mathbf{Y}})^T \tag{22}$$

It is clear that the basic UT algorithm is essentially very simple and easy to apply. For more details and extensions of the UT method, interested readers are referred to [21].

4. Possibilistic uncertainty modeling

The use of classical probability theory in the power systems is faced with two problems including: firstly, the power systems are not closed but open systems. It means that any exterior parameter may influence the system parameters. Secondly, selecting appropriate PDF for uncertain variables is not such an easy task, especially when the available data is inadequate or imprecise [25]. In this condition, the possibility theory may be an encouraging alternative.

4.1. Possibility theory

In possibility theory, for each uncertain value i.e. $\tilde{X}$, the possibility distribution $\pi_{\tilde{X}}(x)$ is used to model the epistemic uncertainty. It defines that how much each element x of universe of discourse U belongs to $\tilde{X}$. It must be recognized that $\pi_{\tilde{X}}(x)=0$, the membership degree of 0, means that x is an impossible event and consequently $\pi_{\tilde{X}}(x)=1$, the membership degree of 1, means that x is a possible event and may be occurred. In possibility theory, the definition of possibility bounds namely the possibility and necessity measures is based on the possibility function. The possibility measure of an event A i.e. $Pos(A)$ is defined as

$$Pos(A) = \sup_{\{x \in A\}} \pi_{\tilde{X}}(x) \tag{23}$$

Consequently, the necessity measure of that event i.e. $Nec(A)$ is defined as

$$Nec(A) = 1 - Pos(not\ A) = 1 - \sup_{\{x \notin A\}} \pi_{\tilde{X}}(x) \tag{24}$$

Note that the notations inf (.) and sup (.) define the lower and upper limits of a given interval under study, respectively.

Various types of membership functions can be used to describe the membership degrees of possibilistic uncertain variables. Here, fuzzy trapezoidal numbers (FTN) with a notation $\tilde{A}=[a_{\min}, a_l, a_u, a_{\max}]$ are used as shown in Fig. 2.

4.2. α-Cut method

In engineering systems, the possibilistic output variable $\tilde{Y}$ of a model of epistemic uncertain variables $\tilde{X}$ is usually represented in the form of a multivariate function $\tilde{Y}=h(\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_N)$. In possibilistic uncertainty analysis, the goal is to find the membership function of output variables when the membership functions of input variables are known. If the possibility distributions of the uncertain input variable $\tilde{X}$ are known, the possibility distribution of $\tilde{Y}$ can be obtained by means of $\alpha-cut$ method. For a given input variable $\tilde{X}$, the $\alpha-cut$ of $\tilde{X}$ is defined as

$$A^\alpha = \{x \in U | \pi_{\tilde{X}}(x) \geq \alpha, 0 \leq \alpha \leq 1\} \tag{25}$$

$$A^\alpha = [\underline{A}^\alpha, A^{-\alpha}] \tag{26}$$

Stress again that U is the universe of discourse of $\tilde{X}$ (i.e. the range of its possible values), $\underline{A}^\alpha$ and $A^{-\alpha}$ are the lower and upper

limits of the A^{α}, respectively. A typical $\alpha-cut$ of a trapezoidal membership function is depicted in Fig. 2. Having the $\alpha-cut$ of each uncertain input variable, the $\alpha-cut$ of output variable $\tilde{Y}$ is calculated as

$$Y^{\alpha}=[\underline{Y}^{\alpha},\ Y^{-\alpha}] \tag{27}$$

$$Y^{\alpha}_{-}=\inf[h(F^{\alpha}_{\tilde{X}_1},\ F^{\alpha}_{\tilde{X}_2},\ldots,F^{\alpha}_{\tilde{X}_N})] \tag{28}$$

$$Y^{-\alpha}=\sup[h(F^{\alpha}_{\tilde{X}_1},\ F^{\alpha}_{\tilde{X}_2},\ldots,\ F^{\alpha}_{\tilde{X}_N})] \tag{29}$$

where, $F^{\alpha}_{\tilde{X}_i}$ stands for the $\alpha-cut$ of the ith possibilistic input variable.

4.3. Defuzzification

The defuzzification converts a fuzzy number into a crisp one [17]. This process can be done by centroid method. The defuzzied value of a given fuzzy quantity i.e. $\tilde{X}$ is calculated as

$$X^{*}=\frac{\int \pi_{\tilde{X}}(x)x\,dx}{\int \pi_{\tilde{X}}(x)dx} \tag{30}$$

5. Uncertainty in the PF problem

5.1. Uncertain parameters

Nowadays, the power industry faces with crucial operational and planning challenges arisen from the power industry restructure accelerationon one hand and the unrivaled advancement of modern technologies during the last decades on the other. The PF study is of the most essential tools for both system operation and planning. However, the uncertainty does exist in many system parameters, which can introduce notable errors in the PF solutions if deterministic data are used. This work deals with UPF studies taking into account the uncertainties associated with the load, wind, and solar power generation as probabilistic uncertainties and the gas turbine DG and EV profiles as possibilistic ones.

5.2. Uncertainty modeling

The load level of a power system is of the most dominant uncertain factors. Generally speaking,different factors such as the time, season, weather conditions, and electricity price, to name a few, can influence the system load [23]. It is usual to describe the load uncertainty with a probabilistic normal distribution function whose parameters, namely its mean and STD values, are calculated according to historical load data [26,27]. Here, we model the load by a normal PDF with mean values equal to the base loads, and STD values equal to a specific percentage of the mean values e.g. 5% [28]. Meanwhile, the power generation of DG units is another uncertain variable of the power grid. As mentioned before, the integration of DG units in distribution network has many technical, socio-economical and environmental benefits. A DG may be controllable such as gas turbine units or may be renewable such as wind and solar farms. DG units are generally non-dispatchable due to their small size and sparsity. Hence, as another uncertain variable, it is assumed that a number of wind farms with uncertain output power are integrated to some buses. Wind speed that determines its power changes both in time and place and its uncertainty is probabilistically modeled by a weibull PDF [29]. To model the wind power uncertainty, the first task is to generate wind speed samples. Then these samples are transformed to wind turbine generator (WTG) output power using wind speed–power curve expressed as

$$P_{WTG}=\begin{cases}0 & \text{if } v\le V_{cut-in} \text{ or } v\ge V_{cut-out}\\ P_{rated}\frac{v-V_{cut-in}}{V_{rated}-V_{cut-in}} & \text{if } V_{cut-in}<v<V_{rated}\\ P_{rated} & \text{if } V_{rated}\le v<V_{cut-out}\end{cases} \tag{31}$$

For sake of simplicity, it is assumed that the power factor of wind farms is kept at 0.85 lag.

Unlike it seems, the solar radiation has a high degree of uncertainty. It varies as a function of several factors such as environmental conditions, time of day, month, season, and orientation of the solar cell generator (SCG) to the sun radiation among the rest. The solar radiation PDF is modeled through the beta distribution function [30]. The beta distribution function is represented as

$$f(R:\alpha_\beta,\ \beta_\beta)=\frac{\Gamma(\alpha_\beta+\beta_\beta)}{\Gamma(\alpha_\beta)\Gamma(\beta_\beta)}R^{\alpha_\beta-1}(1-R)^{\beta_\beta} \tag{32}$$

SCG's output power is related to the solar radiation; therefore, its output power modeling requires the solar radiation modeling. The SCG's output power as a function of radiation is stated as radiation–power curve [31]:

$$P_{SCG}=\begin{cases}P_{rs}\left(\frac{R^2}{R_{STD}R_C}\right) & \text{if} \quad 0\le R<R_C\\ P_{rs}\frac{R}{R_{STD}} & \text{if} \quad R_C\le R<R_{STD}\\ P_{rs} & \text{if} \quad R_{STD}\le R\end{cases} \tag{33}$$

The procedure of uncertainty modeling for solar generation would be the same as that of the wind generation.

The power that a controllable DG unit injects to the power network is uncertain and usually depends on the decision of the DG owner, so the distribution network operator (DNO) cannot have a PDF of it if there is not enough historical data about it and it may be represented possibilistically. In this work, it is assumed that controllable DGs are gas turbine generators and are represented by a trapezoidal membership function such as

$$P_i^{DG}=C_i^{DG}[P_{\min},\ P_l,\ P_u,\ P_{\max}] \tag{34}$$

Deregulation and privatization of power systems changes the way modern power systems all over the world being operated. As a simple instance, in the near future, electric vehicles (EVs) may be of the most important DG technologies due to their soaring number [32]. The EVs' power profile may be negative, zero, or positive since they have battery storages that can charge, discharge, or hold the power [30]. In our work, groups of EVs are considered connected to the system at some buses as parking lots. For sake of simplicity and without loss of generality, their power profiles are aggregated as compound demands, generations or storages [30]; on the other hand, the aggregation points may represent the charging stations or car parkings. Each EV can have three output power states possible: charging ($P_{EV}<0$), no-action ($P_{EV}=0$), and discharging ($P_{EV}>0$). Unlike wind generation, EVs power outputs are essentially functions of their drivers' activity. Due to privacy and security issues, it is not an easy task to access the accurate operation data of each EV, so the estimation of their model parameters is based on expert judgments and some engineering guesses which has no guarantee to be accurate. As a result, the possibilistic distribution can be used to model the uncertainty of EVs' power. The power profiles of our EVs are assumed to have trapezoidal membership functions such as

$$P_i^{EV}=C_i^{EV}[P_{\min},\ P_l,\ P_u,\ P_{\max}] \tag{35}$$

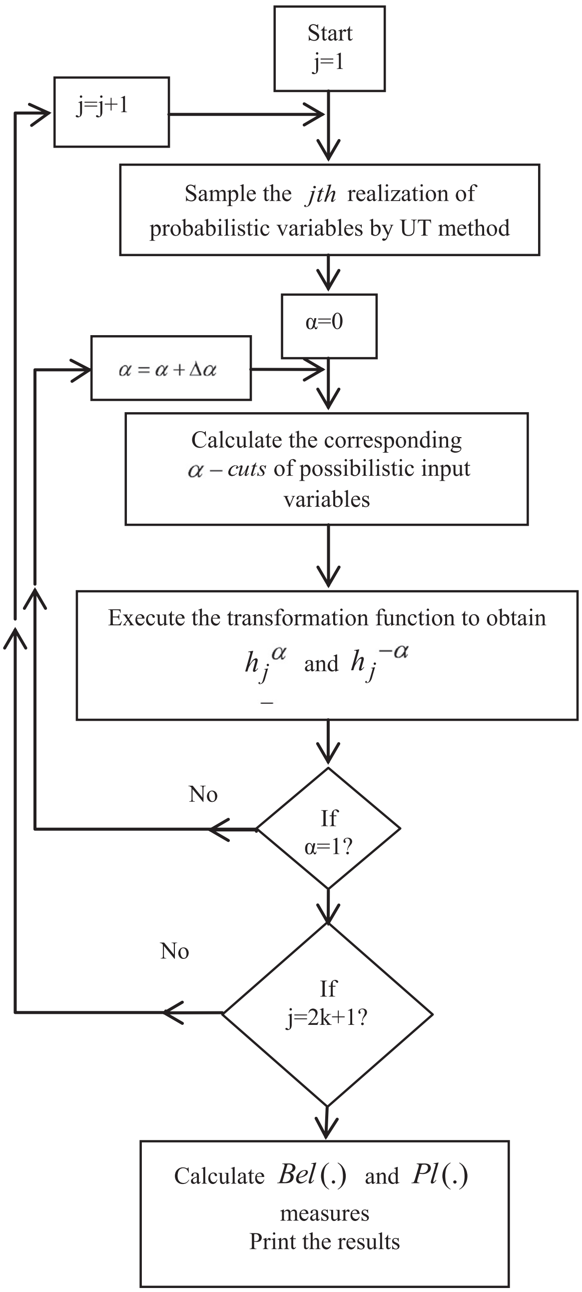

Fig. 3. Flowchart of the proposed algorithm.

6. Evidence theory and joint possibilistic–probabilistic uncertainty modeling

In real world, the uncertainty does exist in all parameters of a given system. The system parameters are influenced by this uncertainty, so it is not a negligible issue and must be taken into account in system performance studies. Performance assessment of an uncertain system is more onerous than a deterministic one. This difficulty escalates when there is incomplete data about some variables i.e. some uncertain variables are probabilistic and some are possibilistic. In this work, a joint possibilistic–probabilistic uncertainty modeling is proposed for this situation.

6.1. Basic idea

In probability theory, a probability density or probability mass, depending on variables being continues or discrete, is assigned to each probable value of a variable. Where on the contrary, this procedure does not hold for evidence theory. In this theory, a given variable X has some subsets as its values, for which positive probability masses are allocated [20]. Then, a mass distribution $(v_i)_{i=1,\dots,k}$ can be defined on all the subsets by assigning each mass value (v_i) to the commensurate subset E_i. The mass distribution must fulfill the condition that

$$\sum_{i=1}^{k} v_i = 1 \tag{36}$$

In this theory, two distinct indices namely the belief $Bel(F)$ and the plausibility $Pl(F)$ functions are used to represent the uncertainty of a set F [30]. Mathematically, $Bel(F)$ and $Pl(F)$ are formulated as follows:

$$Bel(F) = \sum_{E_i, E_i \subseteq F} v(E_i), \quad Pl(F) = \sum_{E_i, E_i \cap F \neq \phi} v(E_i) = 1 - Bel(\overline{F}) \tag{37}$$

Note that the mass distribution of events that insists on F are accounted for $Bel(F)$, while on the contrary, the mass distribution of events that is not in contradiction with F are considered for $Pl(F)$. It means that if we define the interval $[Bel(F), Pl(F)]$, all probability values induced by the mass distribution $v(E)$ on the subset E can be placed in this interval. On the other hand, $Bel(F)$ and $Pl(F)$ can be assumed as the probability lower and upper bounds, respectively.

6.2. Algorithm for joint propagation of probabilistic and possibilistic uncertainties

Consider a general model (linear or nonlinear transformation function) $Y = h(x_1, \dots, x_k, \tilde{x}_{k+1}, \dots, \tilde{x}_N)$ of N uncertain variables $X_i, i = 1, \dots, N$. Consider the case that the first k variables are probabilistic variables described by probability distributions $(p_{X_1}(x), \dots, p_{X_k}(x))$ and the last $N-k$ variables are possibilistic variables represented by possibility distributions $(\pi_{\tilde{X}\,k+1}(x), \dots, \pi_{\tilde{X}\,N}(x))$. In this situation, we propose the hybrid possibilistic–probabilistic uncertainty assessment by combining the UT with fuzzy set theory analysis [31] through the following two major loops [20]:

Outer loop: Repeating UT sampling to perform the uncertainty analysis of probabilistic variables.

Inner loop: executing fuzzy $\alpha - cuts$ analysis to perform the uncertainty assessment of possibilistic variables.

Note that in the outer loop, any probabilistic sampling method such as the MCS, UT and 2PEM among the rest can be utilized. In this work, the UT method is used as probabilistic sampling method since its execution time is less than that of MCS and its accuracy is higher than that of 2PEM. It has some other interesting features presented in [5,21]. The detailed algorithm for joint possibilistic and probabilistic uncertainty modeling can be described step by step as the following [20,33]:

For $j = 1, \dots, 2k+1$ (the outer loop analyzing probabilistic uncertainty), do Steps 1–3:

1. The jth sampling $(x_1^j, x_2^j, \dots, x_k^j)$ of the probabilistic variable vector $(X_1, X_2, \dots, X_k)$ is generated by the UT method.
2. For $\alpha = 0, \Delta\alpha, 2\Delta\alpha, \dots, 1$ (the inner loop analyzing possibilistic uncertainty; $\Delta\alpha$ is the $\alpha - cuts$ method step size, e.g. $\Delta\alpha = 0.05$)
   2.1. The corresponding $\alpha - cuts$ of possibility distributions $(\pi_{\tilde{X}\,k+1}(x), \dots, \pi_{\tilde{X}\,N}(x))$ as the intervals of the possibilistic variables $((\tilde{X}_{k+1}, \dots, \tilde{X}_N))$ are calculated.
   2.2. The minimum and maximum values of the outputs of the model $h(X_1, \dots, X_k, \tilde{X}_{k+1}, \dots, \tilde{X}_N)$, denoted by h_j^{α} and $h_j^{-\alpha}$, respectively are calculated. In this step, the probabilistic variables are fixed at the sampled values in the first step $(x_1^j, x_2^j, \dots, x_k^j)$ while the possibilistic variables take all

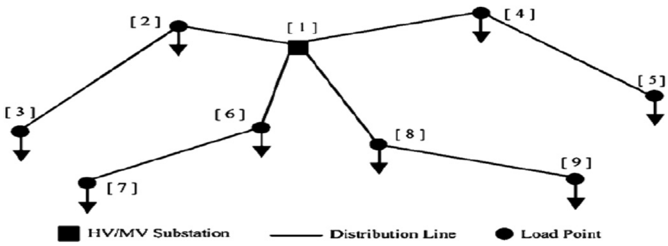

Fig. 4. Single line diagram of the 9-bus distribution network.

values of the $\alpha-cuts$ defined in Step 2.1. Note that here, h is the PF problem.

2.3. The values h_j^{α} and $h_j^{-\alpha}$ representing the lower and upper limits of the $\alpha-cuts$ associated with the output variables $h(x^j{}_1, \ldots, x^j{}_k, X_{k+1}, \ldots, X_N)$ to be saved.
End.

3. The lower and upper limits of different $\alpha-cuts$ of $h(x^i{}_1, \ldots, x^i{}_k, X_{k+1}, \ldots, X_N)$ can be used to approximate the possibility distribution of the model output termed as π_j^Y. It means that fixing the probabilistic variables at jth sampling vector, executing the fuzzy $\alpha-cuts$ method on possibilistic variables, feeding these sample points to the transformation function and saving the values $h_j{}^{\alpha}$ and $h_j{}^{-\alpha}$, the possibility distribution of model output variable can be defined.
End.

Executing the algorithm, results in $2k+1$ possibility distributions $\left(\pi_1^Y, \ldots, \pi_{2k+1}^Y\right)$ approximated for model output. In fact, now we have $2k+1$ possibility distributions of the model output, each obtained from an iteration of the outer loop. For each set E in the universe of discourse of all output values, one can obtain the possibility measure $Pos_j^Y(E)$ and the necessity measure $Nec_j^Y(E)$ using π_j^f as the following:

$$Pos_j^Y(E) = \sup_{\{x \in E\}}\{\pi_j^Y(x)\} \tag{38}$$

$$Nec_j^Y(E) = \inf_{\{x \in E\}}\{1-\pi_j^Y(x)\} \tag{39}$$

Then, these two distinct measures can be used to obtain the belief and plausibility of any set [18].

$$Pl(E) = \sum_{j=1}^{2k+1} p_j\, Pos_j^Y(E) \tag{40}$$

$$Bel(E) = \sum_{j=1}^{2k+1} p_j\, Nec_j^Y(E) \tag{41}$$

Note that p_j is the sampling probability of jth realization $(x_1^j, \ldots, x_k^j)$ of the probabilistic variable vector $(X_1, \ldots, X_k)$ i.e. w^j in the UT method. It is clear that the probability weighted average of the possibility measures for each output fuzzy interval related with each set E is computed by the presented method.

Fig. 3 presents the flowchart of the proposed algorithm. The proposed algorithm has two major steps or on the other hand, it has two different loops, namely the outer and the inner loops. The former considers the uncertainty of probabilistic variables whose task is to sampling of probabilistic uncertain variables. Each iteration of the outer loop has an inner loop, in which the fuzzy interval analysis is carried out. It means that in each inner loop, the samplings of uncertain probabilistic variables are fixed and those of uncertain possibilistic variables are changed to cover the corresponding $\alpha-cuts$ of parameters. When the outer loop finished, the algorithm will be terminated and each output variable would have $2k+1$ membership functions obtained from $2k+1$ iteration of the method. So the membership function of the output

Table 1
Wind farm information-base case.

Parameter	Bus 6
No. of WTGs	2
P_r [MW]	0.6

Table 2
Gas turbine DG unit information-base case.

Parameter [MW]	Value
C_5^{DG}	2
$P_{\min}$	0
P_L	0.9
P_U	1
$P_{\max}$	1

Table 3
Crisp data parameters-base case.

Parameter [MW]	Mean value	STD value
P^{grid}	31.4947	0.6758
Loss	0.8029	0.0229
P^{1-2}	9.5942	0.3769

Table 4
EVs information-case 1.

Parameter	Bus 9
C_i^{EV} [kW]	5
No. of EVs	500
$P_{\min}$ [kW]	−1
P_L [kW]	−0.6
P_U [kW]	0.6
$P_{\max}$ [kW]	1

variables in this situation does not have deterministic parameters. It means that these parameters are themselves probabilistic. Moreover, if all the input variables are probabilistic, then the inner loop will be eliminated and a simple UT method would be remained. If all the variables are possibilistic, the outer loop will be eliminated and therefore, the remainder will be the $\alpha-cuts$ method of the fuzzy sets analysis. In case of having both probabilistic and possibilistic variables, the joint algorithm including both two loops must be implemented accordingly. It must be recognized that for any iteration of the outer loop, the inner loop must be repeated which is a time consuming procedure, so the reduction of outer loop iterations while keeping the accuracy in the desired level is really an interesting issue. So in this work, the UT method whose application is validated in PPF studies in [5] is proposed to reduce the outer loop iterations compared with that of MCS.Worthily to mention that for each outer loop repetitions, the inner loop is repeated $1+1/\Delta\alpha$ times in which a PF problem must be solved. Hence, there is a great incentive to reduce the outer loop repetitions while keeping the accuracy in the desired level.

6.3. Probabilistic propagation

For pure probabilistic propagation, the possibilistic distributions must be converted into probability density functions. This conversion can be done using various techniques [34], i.e. in this

paper by simple normalization:

$$p_{X_i}(x) = \frac{\pi_{\tilde{X}_i}(x)}{\int_0^{+\infty} \pi_{\tilde{X}_i}(x)dx} \tag{42}$$

When the probabilistic distribution for each fuzzy variable is determined, only the outer loop of the algorithm is performed $2N+1$ times. This means that in this case, the algorithm will have only one loop, the outer loop. In each iteration of that loop, the vector $(X_1, X_2, \ldots, X_N)$ is sampled and the corresponding output variables are calculated. Performing $2N+1$ repetitions of the outer loop, the probability distributions of system parameters can be obtained or on the other hand, in this situation, all uncertain parameters are represented probabilistically.

7. Simulation studies

In order to justify the effectiveness of the proposed methodology, it is examined through two dimensionally different distribution networks. The proposed method was implemented on a personal computer with a 2.8-GHz processor and 4-GB of RAM using Mathpower simulation package [35].

7.1. The 9-bus distribution test system

A 9-bus distribution system is examined in order to show the performance of the proposed methodology in small scale distribution systems. The single line diagram of this system is depicted in Fig. 4 and the technical data of this network is taken from [36,37]. The network consists of a 132/33 kV substation with 40 MVA capacity and has 8 feeders with eight aggregated loads.

7.1.1. The base case

In the base case, it is assumed that the load as a probabilistic uncertain variable is modeled by a normal PDF with the mean value equal to the quantities declared in [36,37] and the STD value equal to 5% of the mean values. The wind generation and gas turbine DGs are considered as other probabilistic and possibilistic variables, respectively. Table 1 presents the data about wind farm of this case. In this case, it is assumed that a gas turbine DG unit with installed capacity of 2 MW and a fuzzy membership function such as one presented in Table 2 is installed at bus 2 of the network. It must be recognized that in this case, the total DG penetration level is 10% of total demand.

The total imported power from the main grid, total network losses and the power transmitted through the line between buses 1 and 2 are considered as output variables. Table 3 presents the mean and STD of crisp values for these output variables.

7.1.2. Case 1: embedding EVs to distribution networks

The intention at this point is to interrogate the effects of EV aggregation on distribution networks. So, assume the case in which the uncertain variables are the same as those of the base case except that a 2.5 MW EVs block comprised of 500 individual EVs is connected to bus 9 of the system. The fuzzy membership function of each EV is considered the same as Table 4. Table 5 presents presents the probabilistic data of crisp values for the output variables. Note that the values in Table 5 are almost the same as those of Table 3 due to the symmetry of considered membership function for EVs' power profile.

Fig. 5 compares the plausibility function and pure probabilistic measures of the base case and case 1. The belief function is not plotted to avoid figures crowding. It is clear that in case 1 for which, the EV uncertainty is added to the network, the gap between plausibility and pure probabilistic measures has been increased severely. Note that the pure probabilistic measure has shifted down while the plausibility measure function has shifted up compared to the base case. These shifts are due to the uncertainty of EVs' profile.

7.1.3. Case 2: DG penetration increase

Assume the case in which the DG penetration level has been increased to 30% of total demand by doubling the installed capacity of all DG units of the case 1. Table 6 presents the obtained

Table 5
Crisp data parameters case 1.

Parameter [MW]	**Mean value**	**STD value**
P^{grid}	31.5089	0.6506
Loss	0.8172	0.0221
P^{1-2}	9.5982	0.376

Table 6
Crisp data parameters case 2.

Parameter [MW]	**Mean value**	**STD value**
P^{grid}	31.2010	0.8978
Loss	0.8504	0.028
P^{1-2}	9.6047	0.3781

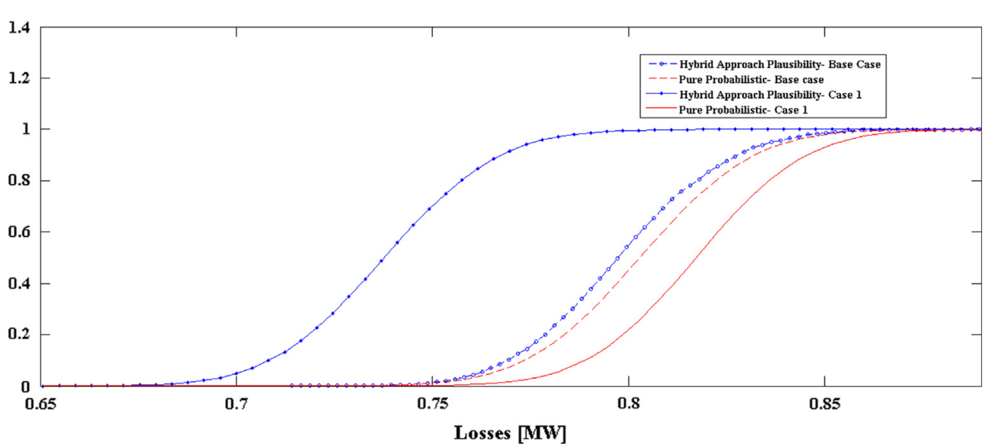

Fig. 5. Comparison of the plausibility function and pure probabilistic measures of the base case and case 1.

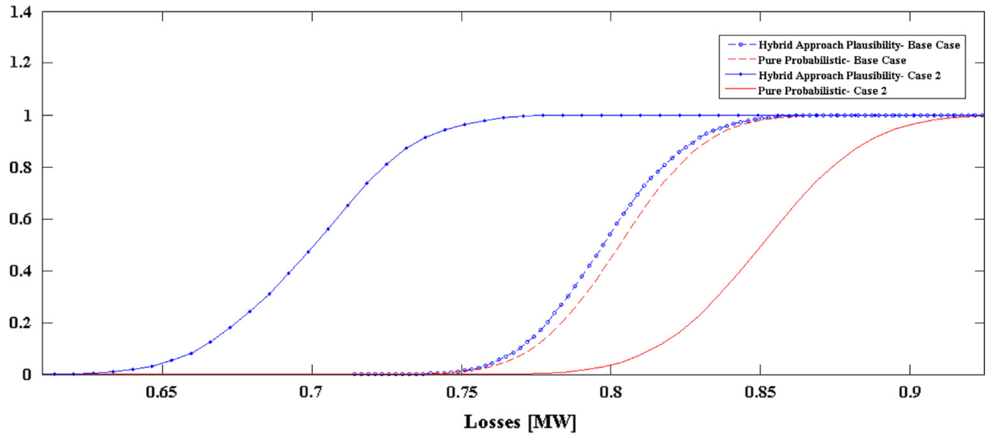

Fig. 6. Comparison of the plausibility function and pure probabilistic measures of the base case and case 2.

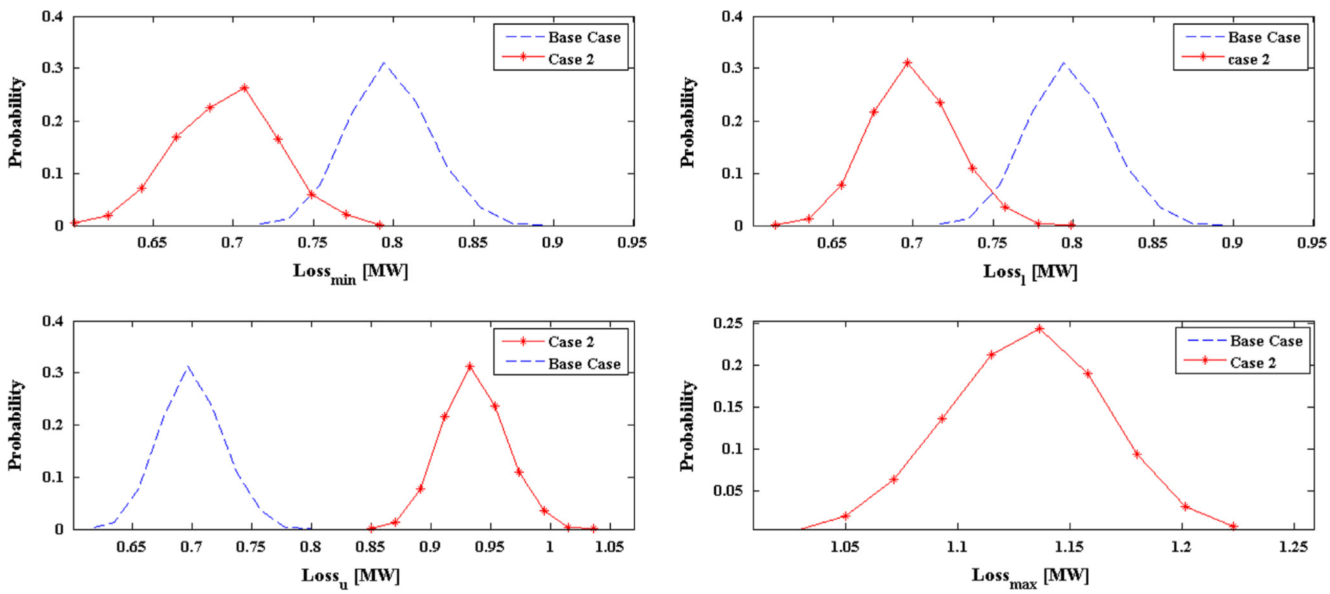

Fig. 7. The PDF of membership function parameters of the losses for the base case and case 2.

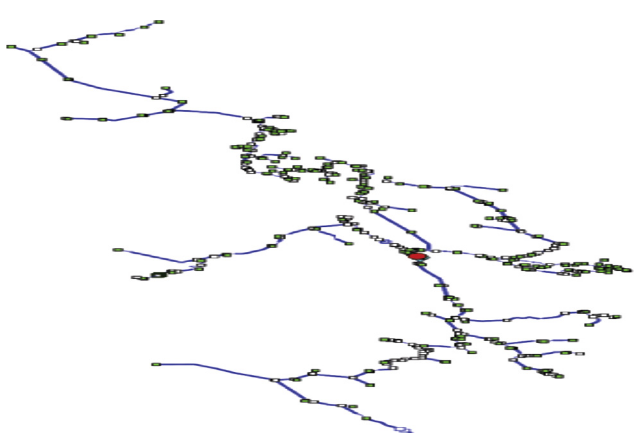

Fig. 8. The realistic 574-node distribution system.

Table 7
Wind farm information-574 node system.

Parameter	Bus 90
No. of WTGs	5
P_{rw} [MW]	0.1
C_w [m/sec]	8
K_W	3

Table 8
Solar farm information-574 node system.

Parameter	Bus 541
P_{rs} [MW]	0.3
α_β [kW/m^2]	0.5
β_β[kW/m^2]	0.3

results for this case. Comparing the results of this table and those of Table 3, the STD value of all output variables have been increased due to a higher uncertainty degree. Fig. 6 compares the plausibility function and pure probabilistic measures for the total active losses of the base case and case 2. It is clear that higher penetration level caused the gap between these two representations to increase, dramatically. Fig. 7 compares the PDF of membership function parameters for the losses in the base case and case 2. It can be seen that the PDF of $Loss_{\min}$ and $Loss_l$ have been shifted to the left, the PDF of $Loss_u$ has been shifted to the right and the PDF of $Loss_{\max}$ is unchanged since the DG penetration increase can decrease the minimum of losses. Shifting

the $Loss_u$ to the right is due to the fact that EVs can either being generators or consumers.

Comparing the results of Tables 3 and 6 reveals that the mean value of the total losses has been increased in case 2 that is confirmed by Fig. 7, in which, the PDF of $Loss_u$ has been shifted to the right, notably.

7.2. A realistic 574-node distribution system

Here, the proposed methodology is applied on a realistic 574-node distribution network presented in Fig. 8. This network has a 20 MVA capacity substation at node 552 and 180 load points. Detailed data of this network is taken from [38,39]. This network is examined here to demonstrate the performance of the proposed method in real scale distribution networks.

Table 9
Gas turbine DG units information 574 node system.

Parameter	Bus 445
$C^{DG}{}_i$ [MW]	0.5
$P_{\min}$ [p.u.]	0
P_l [p.u.]	0.1
P_u [p.u.]	0.9
$P_{\max}$ [p.u.]	1

Table 10
EVs information-574 node system.

Parameter	Bus 334
No. of EVs	100
$C^{EV}{}_i$ [kW]	5
$P_{\min}$ [p.u.]	−1
P_l [p.u.]	−0.8
P_u [p.u.]	0.6
$P_{\max}$ [p.u.]	1

The load uncertainty is modeled with a normal PDF with 5% STD. Tables 7–10 present the data about uncertain input variables. More detailed information about used wind and solar farms is given in Appendix (Table A.1).

As noted earlier, a way to represent the uncertainty of such a situation is to associate specific membership functions to the output variables and represent its parameters probabilistically. Figs. 9 and 10 show this fact and present the CDF of each parameters of the membership function of total active losses and imported power from the main grid, respectively.

Fig. 11 compares the plausibility and pure probabilistic measures of the losses for this case obtained by the MCS and UT as probabilistic uncertainty analysis methods. In order to compare the results with the past work [17], the plausibility and pure probabilistic measures of output variables obtained by the UT method are compared with those of MCS with a number of 10,000 samples which are considered the most accurate. Fig. 11 confirms that the UT method can be used in the proposed joint uncertainty assessment tool as its accuracy is well within the desired level. The crisp data parameters of this case study are outlined in Table 11. Table 12 compares the execution time of the proposed method and the MCS-based method. The presented results in this table illustrate that the proposed method is computationally efficient and can reduce the computational burden, dramatically. Accordingly, the effectiveness of the proposed method is confirmed through both accuracy and execution time criteria.

8. Conclusion

In this paper, a new approach for uncertain power flow (UPF) studies considering different kinds of uncertainties from probabilistic to possibilistic ones is proposed. Traditional probabilistic or possibilistic methods can take into account just one of these kinds of uncertainties but the proposed method can handle both, simultaneously. The evidence theory is used in joint propagation of possibilistic and probabilistic uncertain variables in a typical

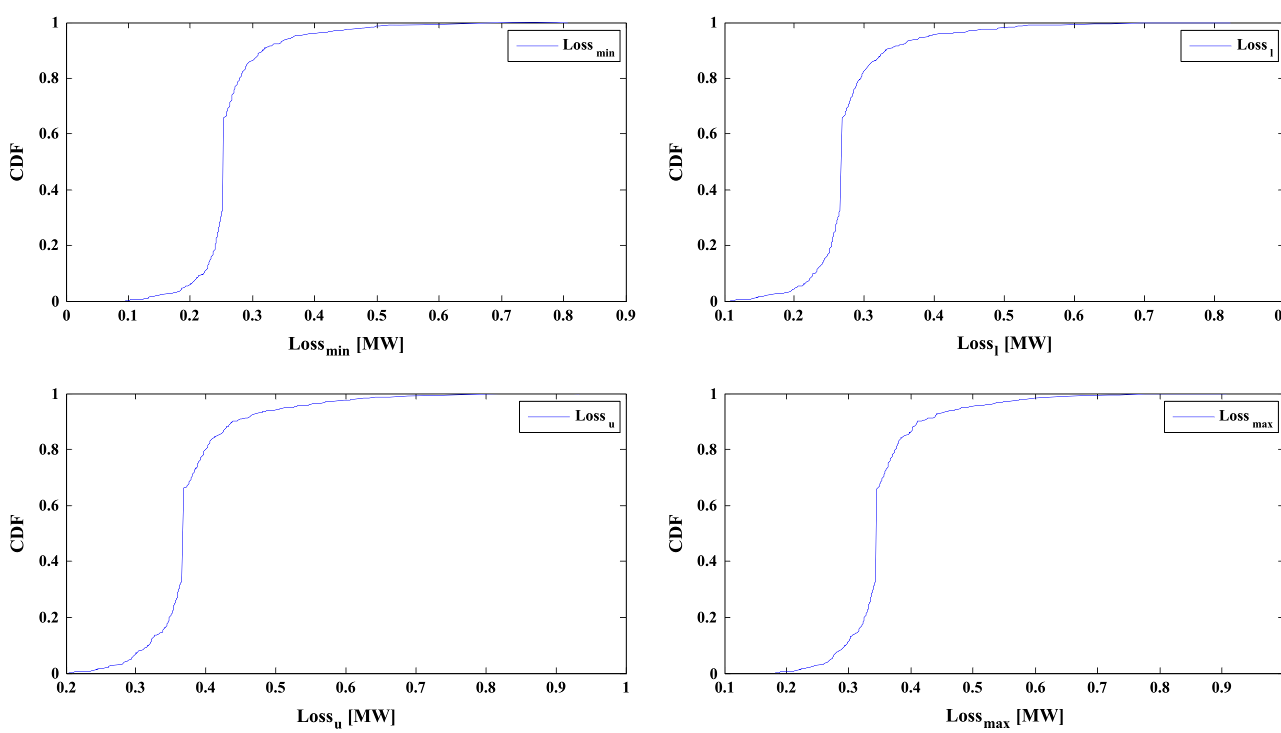

Fig. 9. Probabilistic representation of the total active losses membership function parameters.

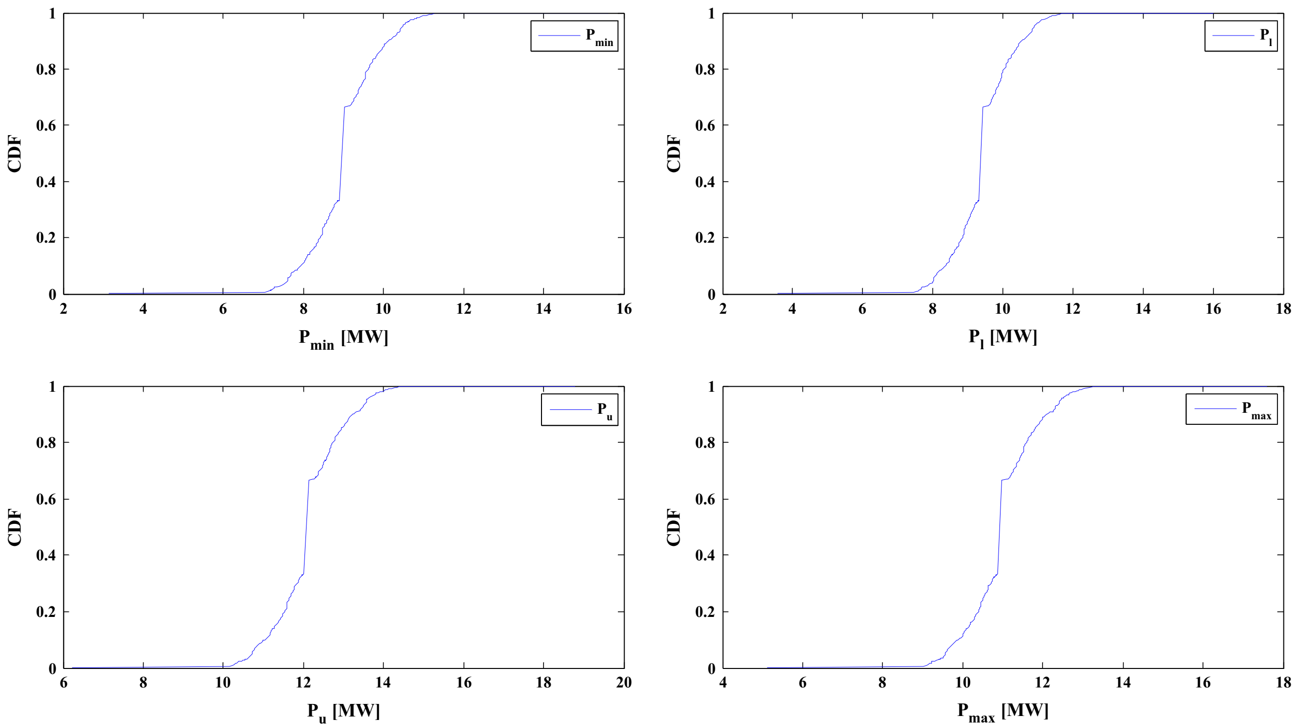

Fig. 10. Probabilistic representation of the power from the main grid membership function parameters.

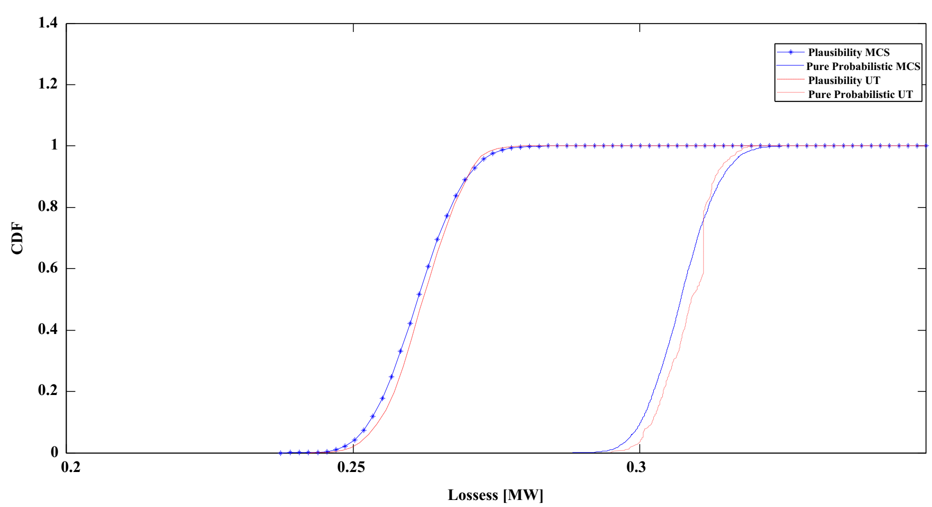

Fig. 11. Comparison of plausibility and pure probabilistic measures of the losses for 574 nod system, obtained by the MCS and UT.

Table 11
Crisp data parameters-574 node system.

Parameter	Mean Value	STD value
P^{G}_{slack} [MW]	10.3393	0.9409
Loss [MW]	0.319	0.0716
P^{1-5} [MW]	0.7873	0.1985

Table 12
Run time comparison- both systems.

Method	Run time [s] 9-bus	Run time [s] 574-bus
Joint method using MCS	8.814e3	1.8514e5
Proposed method	20.27	6.7577e3

problem. Then a novel algorithm is proposed to jointly propagate both of these variables in a PF study problem. Some of uncertain input variables have enough historical data and therefore can be represented by the probability theory i.e. having a specific probability density function (PDF) such as the wind speed, solar radiation, and load pattern in a given geographical zone. Some others are unknown and we have a little data about them or may be obtained by experience which is not accurate. These variables may be represented possibilistically such as the power generated by a private controllable distributed generation (DG) or the state of charge of an individual electric vehicle (EV). In the following, we have examined examples of UPF analysis formulation considering probabilistic and possibilistic uncertainties. Two types of uncertainties (i.e. probabilistic and possibilistic) have been identified in the case studies considered and their representation has been described. Then, the joint propagation of different representations of uncertainties has been illustrated within the frame of evidence

theory, which integrates the results in the form of plausibility and belief functions. Two numerical case studies have been used to demonstrate the effectiveness of the proposed methodology. Simulation results confirm the impressiveness of the proposed method from the view point of accuracy and execution time criteria.

The cumulative distribution function (CDF) of the system output parameters obtained by the pure probabilistic method lies within the belief and plausibility functions obtained by the joint propagation approach. Also, the lack of precision in the DG parameters is explicitly reflected by the gap between the belief and plausibility functions. In addition, the separation between plausibility and belief or between plausibility and pure probabilistic measures, due to the epistemic uncertainty on the DG resources parameters, grows as the penetration level increases. Moreover, the chance from both joint propagation and pure probabilistic methods decreases as the penetration level increases.

Acknowledgment

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Appendix

The detailed information about wind and solar farms is presented in Table A.1.

Table A.1
Detailed wind and solar farms' information.

Parameter		Value
Wind turbine data	V_i [m/s]	3
	V_r [m/s]	12.5
	V_o [m/s]	25
Solar data	R_{std} [W/m^2]	1000
	R_c [W/m^2]	150

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